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GENERAL REGULAR BOUNDARY VALUE PROBLEMS FOR
DEGENERATE HYPERBOLIC EQUATIONS

ABSTRACT

of the PhD thesis for the degree of
doctor of Philosophy (PhD) in the specialty
«6D060100-Mathematics»

The relevance of the research topic. In the modern theory of partial differential equations, an important place is occupied by the study of degenerate hyperbolic and elliptic equations, as well as equations of mixed type. The increased interest in this class of equations is explained both by the great theoretical significance of the results obtained and by their numerous applications in gas dynamics, hydrodynamics, in the theory of infinitesimal surface bendings, in the momentless theory of shells, in various branches of continuum mechanics, acoustics, and in the theory of electron scattering. and many other fields of knowledge. The development of modern science has shown that degenerate equations are a good model of real physical and biological processes. And this led to the relevance of setting and solving various boundary value problems for them, which are currently the subject of fundamental research by many mathematicians.

After the fundamental works of F. Tricomi and S. Gellerstedt, important results in the study of differential equations of the type under consideration are given in the well-known monographs by A. V. Bitsadze and M. M. Smirnov.

The solution of the main Tricomi boundary value problem and other boundary value problems was carried out using the methods of complex singular integral equations. Therefore, the problem was posed to find a correct boundary value problem, which is easier to solve and with the help of which other boundary value problems are solved.

The variety of well-posed boundary value problems in the domains contained in the characteristic cone is much wider than in the standard mixed cylindrical domain, but they can be solved only for a narrow class of equations.

In the study of the mixed Cauchy problem in a cylindrical domain, the lateral boundary conditions were, as a rule, local boundary conditions of the Dirichlet type, or periodic boundary conditions.

In the work of Kalmenov T.Sh., Suragan D. for the first time found the boundary condition of the Newtonian (volumetric) potential, which is a new integro-

differential self-adjoint boundary condition for the Laplace equation. It is of interest to study the solution of the mixed Cauchy problem for noncharacteristic degenerate hyperbolic equations imposed by this boundary condition.

Degenerate hyperbolic equations have the peculiarity that the well-posedness of the Cauchy problem does not always hold. The Cauchy problem in the usual formulation may turn out to be unsolvable if the hyperbolic equation degenerates along a line that is simultaneously a characteristic (envelope of a family of characteristics), or if the coefficients of the hyperbolic equation at lower terms are singular. Therefore, it is natural to consider a "modified" Cauchy problem, when the initial conditions on the line of parabolic degeneracy are given with weight functions.

The relevance of the chosen research topic is confirmed by the rapid development of research in this direction, as well as the inclusion of numerous publications on expressive differential equations in the authoritative international databases Web of Science, Scopus, MathSciNet, etc.

The aim of the PhD thesis Finding a solution to the mixed Cauchy problem for noncharacteristic degenerate hyperbolic equations in the classical Sobolev space and finding common regular boundary conditions for ordinary differential equations that are analogs of degenerate hyperbolic equations.

To achieve the aim of the dissertation, **the main objectives** of the following research are considered:

- finding a general boundary condition for ordinary differential equations that are analogs of degenerate hyperbolic equations;
- Consideration of the Cauchy problem, where the initial conditions are given by weight functions, using the boundary condition of the Newtonian (volumetric) potential for a hyperbolic equation of the Euler-Poisson-Darboux type;
- Mixed Cauchy problem for one class of noncharacteristic degenerate hyperbolic equations: 1) mixed Cauchy problem with a side boundary condition for noncharacteristic degenerate hyperbolic equations in the case $a(t) \equiv b(t) \equiv 0$; 2) Studying the general case of the Cauchy problem.

Object of the PhD thesis. Study of the mixed Cauchy problem for non-characteristic degenerate hyperbolic equations using the Newton potential boundary condition, study of the modified Cauchy problem for a hyperbolic equation of the Euler-Poisson-Darboux type and finding general regular boundary value problems for degenerate ordinary differential equations using the theory of regular expansion of minimal operators.

The methods of scientific research. To conduct research on this topic, along with new ideas in mathematics, simple and independently derivable differential

equations, methods of the theory of mathematical physics are used. We also recommend creating and using new methods of original research based on our results.

Scientific novelty of the work. In this PhD thesis, we study the mixed Cauchy problem for one class of noncharacteristic degenerate hyperbolic equations using the Newton potential boundary condition. Unlike other works devoted to this topic, where solutions of the mixed Cauchy problem with different lateral boundary conditions of the problems under consideration are obtained in weighted spaces, in this paper, all solutions of the mixed Cauchy problems under consideration are obtained in classical Sobolev spaces.

Theoretical and practical significance of the results. The research work is mainly theoretical and fundamental. Therefore, the scientific significance of the work is associated with the use of deep, modern results of the theory of well-posedness of initial and boundary value problems and the creation of new proprietary research and analysis methods.

Publications. On the topic of the thesis 6 papers were published, including 4 publications in a high-ranking scientific journal, indexed in the Web of Science and Scopus, 3 publications in scientific journals included in the list recommended by the Committee on the Control of Education and Science of the MES RK for publication of the main scientific results of scientific activities. The results on the topic of the thesis were published in the following papers:

Publication in the high-ranking scientific journals

1 Kakharman N., Kal'menov T., Mixed Cauchy problem with lateral boundary condition for noncharacteristic degenerate hyperbolic equations. *Boundary Value Problems*. 1 (2022): 1-11. (Web of Science Impact factor=1,7(Q1), Scopus SJR=0,573 (Q1), CiteScore=3,5, Scopus Percentile=92).

2 Kakharman N., Otelbaev M., Solution estimates for one class of elliptic and parabolic nonlinear equations. *Complex Variables and Elliptic Equations*. (2022): 1-10. (Web of Science Impact factor=0.765(Q3), Scopus SJR=0,454, CiteScore=1.6; Scopus Percentile=51).

3 Kakharman N., Tulenov K. and Zhumanova L., On hyponormal and dissipative correct extensions and restrictions. *Mathematical Methods in the Applied Sciences*, 2022, 45(16), pp. 9049–9060. (Web of Science Impact factor=3.007(Q1), Scopus SJR=0,702 (Q1), CiteScore=3,9; Scopus Percentile=91).

CCES

1 Kal'menov T. Sh., Kakharman N., On a problem of the Bitsadze-Samarskii for the Sturm-Liouville equation, *Mathematical journal*. -2018, T18, V1.

2 Kal'menov T. Sh., Kakharman N., Sadybekov M.A., About root functions of periodic Sturm-Liouville problem, *Kazakh Mathematical Journal*. -2019, V: 19:1, pp 31-38;

3 Kal'menov T.Sh., Kakharman N., On the completeness of root vectors of regular boundary value problems for one-dimensional differential operators. *Kazakh Mathematical Journal*. -2020. – T.20. - №2. 73-84.

The structure and scope of the thesis. The PhD thesis includes a title page, content, introduction, four chapters, conclusion and list of references, consisting of 55 titles. The total volume of the thesis is 77 pages.

The main content of the thesis. In Chapter 1, using an abstract theorem of the theory of extension and restriction, we find a general regular boundary condition for one-dimensional analogues of degenerate equations - differential equations of the first order.

Further, in the second chapter, the problem of a general regular boundary condition for second-order differential equations was studied. A general boundary condition is found for weakly degenerate equations. First, consider the Cauchy problems with the Newtonian (volume) potential boundary condition for the equation

$$Lu = \frac{\partial}{\partial t} \left(t^\beta \frac{\partial}{\partial t} u \right) - \Delta_x u = f(x, t)$$

It is known that this equation can be reduced to the Euler-Poisson-Darboux equation.

Last third chapter: consider the mixed Cauchy problem for a class of non-characteristic degenerate hyperbolic equations using the Newtonian (volumetric) potential boundary condition

$$Lu = u_{tt} - k(t)\Delta_x u + b(t) \frac{\partial u}{\partial t} + a(t)u = f(x, t),$$

$$u|_{t=0} = 0, \quad \frac{\partial u}{\partial t} \Big|_{t=0} = 0$$

$$N[u] \equiv -\frac{u(x,t)}{2} + \int_{\partial\Omega} \left(\frac{\partial \varepsilon}{\partial n_\xi}(x, \xi) \cdot u(\xi, t) - \varepsilon(x, \xi) \cdot \frac{\partial u}{\partial n_\xi}(\xi, t) \right) d\xi = 0, \\ 0 < t < T, \quad x \in \partial\Omega,$$

where $k \in C^{1+\alpha}[0, T]$, $0 < \alpha < 1$, $k(t) > 0$, $t > 0$, $k(0) = 0$, $k'(t) \geq 0$, и $\varepsilon(x, \xi)$ is a fundamental solution of the Laplace equation.

Unlike other works devoted to this topic, where solutions of the problems under consideration are usually obtained in weighted spaces, in this work the solution is obtained in classical Sobolev spaces, including for the main part of the equation.

In the final part, we summarize the results of the study and suggest directions for further development.